Reasoning, Lines, and Transformations

G.1 a-d	1.___ 2.___ 3.___ 4.___ 5.___ 6.___ 7.___
G.2 a-c	1.___ 2.___ 3.___ 4.___ 5.___ 6.___
G.3 a-d	1.___ 2.___ 3.___ 4.___ 5.___ 6.___
G.4 a-g	1.___ 2.___ 3.___ 4.___ 5.___ *(6 & 7 in packet)* 8.___

SOL G.1

The student will construct and judge the validity of a logical argument consisting of a set of premises and a conclusion. This will include a) identifying the converse, inverse, and contrapositive of a conditional statement; b) translating a short verbal argument into symbolic form; c) using Venn diagrams to represent set relationships; and d) using deductive reasoning, including the law of syllogism.

HINTS & NOTES

Conditional:
If p, then q.
p → q

Inverse: Insert "nots"
If not p, then not q.
~p → ~q

Converse: Change order
If q, then p.
q→ p

Contrapositive: Change order and insert "nots"
~q → ~p

Symbols:
~ not
∧ and
∨ or
→ if then, implies
↔ if and only if
∴ therefore

Biconditional:
p if and only if q.
p↔q

Law of Syllogism:
If a→b and b→c, then a→c.

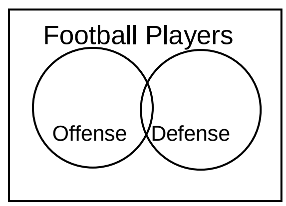
Law of Detachment:
If a→b is true.
a is true.
b is true.

PRACTICE G.1

1. **Which is the converse of the sentence, "If Sam leaves, then I will stay."?**

   A. If I stay, then Sam will leave.
   B. If Sam does not leave, then I will not stay.
   C. If Sam leaves then I will not stay.
   D. If I do not stay, then Sam will not leave.

2. **According to the Venn diagram, which is true?**



   F. Some football players play offense and defense.
   G. All football players play defense.
   H. No football players play offense and defense.
   J. All football players play offense or defense.

3. **Let *a* represent "x is an even number."**
   Let *b* represent "x is a multiple of 4."
   When x = 10, which of the following is true?

   A. $a \wedge b$
   B. $a \wedge \sim b$
   C. $\sim a \wedge b$
   D. $\sim a \wedge \sim b$

4. **Which statement is the inverse of "If the waves are small, I do not go surfing"?**

   A. If the waves are not small, I do not go surfing.
   B. If I do not go surfing, the waves are small.
   C. If I go surfing, the waves are not small
   D. If the waves are not small, I go surfing.

HINTS & NOTES

The "if-then" statement always has the same truth value as its contrapositive. That is, they are logically equivalent.

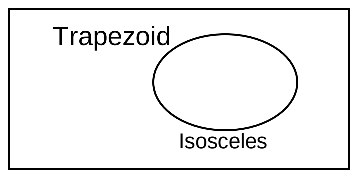
PRACTICE G.1

5. **"If negotiations fail, the baseball strike will not end."**

   "If the baseball strike does not end, the World Series will not be played."

   F. If the baseball strike ends, the World Series will not be played.
   G. If negotiations do not fail, the baseball strike will not end.
   H. If negotiations fail, the World Series will not be played.
   J. If negotiations fail, the World Series will be played.

6. **According to the Venn diagram, which statement is true –**



   A. No trapezoids are isosceles trapezoids.
   B. Some trapezoids are isosceles trapezoids.
   C. All trapezoids are isosceles trapezoids.
   D. Some isosceles trapezoids are parallelograms.

7. **Which statement is logically equivalent to *"If it is warm, then I go swimming"***

   F. If I go swimming, then it is warm.
   G. If it is warm, then I do not go swimming.
   H. If I do not go swimming, then it is not warm.
   J. If it is not warm, then I do not go swimming.

SKILLS CHECKLIST: *I can…*

- ☐ Identify the converse, inverse and contrapositive of a conditional statement.
- ☐ Translate verbal arguments into symbolic form.
- ☐ Determine the validity of a logical argument.
- ☐ Use deductive reasoning (Law of Syllogism, Law of Detachment, Counterexamples)
- ☐ Select and use methods of reasoning and proofs.
- ☐ Interpret Venn Diagrams and represent set relationships.
- ☐ Use and recognize symbols of logic including $\rightarrow$, $\leftrightarrow$, ~, $\therefore$, $\wedge$, and $\vee$.

SOL G.2

The student will use the relationships between angles formed by two lines cut by a transversal to a) determine whether two lines are parallel; b) verify the parallelism, using algebraic and coordinate methods as well as deductive proofs; and c) solve real-world problems involving angles formed when parallel lines are cut by a transversal.

HINTS & NOTES

To prove that 2 lines are parallel, you must be able to prove:

-corresponding angles are congruent
OR
-alternate interior angles are congruent
OR
-alternate exterior angles are congruent
OR
-consecutive interior angles are supplementary
OR
-both lines are perpendicular to the same line
OR
-both lines are parallel to the same line
OR
-the two lines have the same slope.

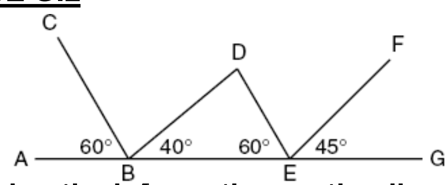
Slope = $\dfrac{y_2 - y_1}{x_2 - x_1}$

Parallel lines – have the same identical slope

Perpendicular lines – have negative reciprocal slopes

PRACTICE G.2

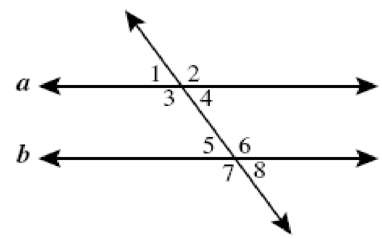
1.



Using the information on the diagram, which is true?

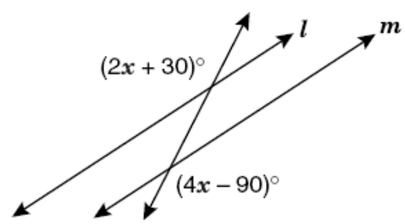
A. $\overline{BD} \parallel \overline{EF}$
B. $\overline{BD} \parallel \overline{DE}$
C. $\overline{CB} \parallel \overline{BD}$
D. $\overline{CB} \parallel \overline{DE}$

2. Line *a* is parallel to line *b* if -



F. $\angle 1 \cong \angle 4$
G. $\angle 1 \cong \angle 7$
H. $\angle 1 \cong \angle 6$
J. $\angle 1 \cong \angle 5$

3.



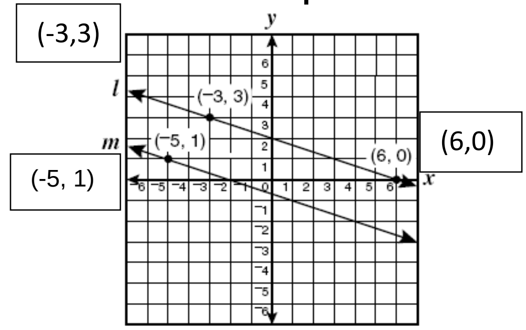
What value for x will show that lines *l* and *m* are parallel??

A. 60
B. 40
C. 30
D. 25

HINTS & NOTES

PRACTICE G.2

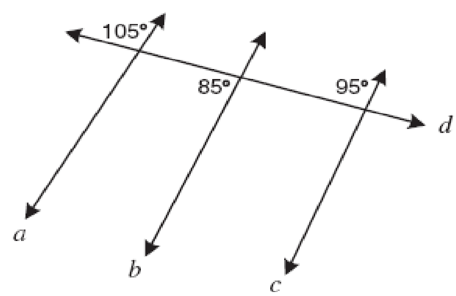
4. **Lines *l* and *m* contain the points shown.**



Which of the following points *must* lie on line *m* in order for lines *l* and *m* to be parallel?

- **F.** (0, -2)
- **G.** (0, -1)
- **H.** (1, -1)
- **J.** (4, -1)

5. **In this diagram, line *d* cuts three lines to form the angles shown.**



Which two lines are parallel?

- **A.** *a* and *b*
- **B.** *a* and *c*
- **C.** *b* and *c*
- **D.** No lines are parallel

6. **Which statement describes the lines whose equations are y= 1/3x + 12 and 6y = 2x + 6 ?**

- **F.** They are segments**.**
- **G.** They are perpendicular to each other.
- **H.** They intersect.
- **J.** They are parallel to each other.

SKILLS CHECKLIST: *I can…*

- ☐ Verify if two lines are parallel using deductive, algebraic and coordinate proof.
- ☐ Solve problems involving intersecting lines and transversals.
- ☐ Solve real-world problems involving intersecting and parallel lines in a plane.

SOL G.3

The student will use pictorial representations, including computer software, constructions, and coordinate methods, to solve problems involving symmetry and transformation. This will include a) investigating and using formulas for finding distance, midpoint, and slope; b) investigating symmetry and determining whether a figure is symmetric with respect to a line or a point; and c) determining whether a figure has been translated, reflected, or rotated.

HINTS & NOTES

Midpoint: $(\frac{x_2+x_1}{2}, \frac{y_2+y_1}{2})$

Slope: $\frac{y_2-y_1}{x_2-x_1}$

Distance: $\sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$

Parallel lines have the same slope.

Perpendicular lines have a negative, reciprocal slope.

Transformations

Reflection – flips over

Translation – slides over

Rotation – turns around a point

Dilation – shrinks or blows up

A **line of symmetry** is a line that can cut an object so that if it is folded all sides and angles will match perfectly.

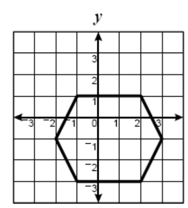
PRACTICE G.3

1. **What is the slope of the line through (-2, 3) and (1, 1)?**

   A. $-\frac{3}{2}$ **B.** $-\frac{2}{3}$

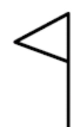
   C. $\frac{1}{2}$ **D.** 2

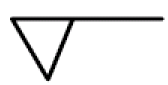
2. **The hexagon in the drawing has a line of symmetry through –**

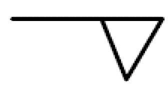


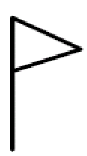
   F. (-1. -3) and (2, 1)
   G. (1, 1) and (1, -3)
   H. (2, 3) and (2, -3)
   J. (-2, -1) and (3, -1)

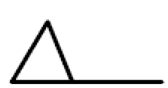
3. **Consider the figure provided. Which of the following is a rotation in the plane of the given figure?**



A 

B 

C 

D 

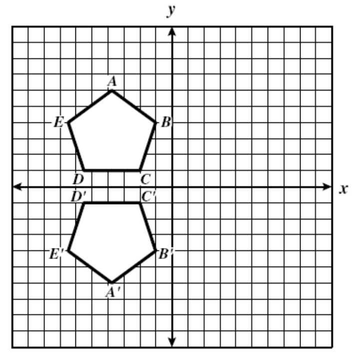
HINTS & NOTES

Point Symmetry occurs when an object can be rotated 180° and mapped onto itself.

If you turn the figure upside down, does it look the same?

PRACTICE G.3

4. The polygon A'B'C'D'E' is -?



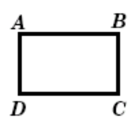
F. a translation of ABCDE across the x- axis
G. a 180^{o} clockwise rotation of ABCDE about the origin
H. a reflection of ABCDE across the y-axis
J. a reflection of ABCDE across the x-axis

5. Which point is the greatest distance from the origin?

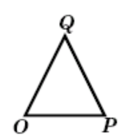
A. (9, 2)
B. (3, 4)
C. (-9, 1)
D. (-8, -5

6.

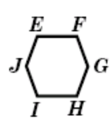
F.



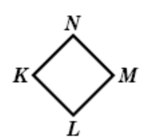
G.



H.



J.



Which polygon shown above has only one line of symmetry?

SKILLS CHECKLIST: *I can…*

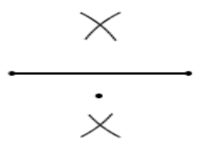
- ☐ Use midpoint formula to find the coordinates of the midpoint of a segment.
- ☐ Use a formula to find the slope of a line.
- ☐ Compare slopes to determine whether two lines are parallel, perpendicular, or neither.
- ☐ Determine if a figure has point symmetry, line symmetry, both or neither.
- ☐ Identify what type of transformation has taken place (reflection, rotation, dilation, or translation.)
- ☐ Use distance formula to find the length of a line segment given coordinates of endpoints.

SOL G.4

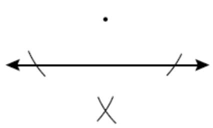
The student will construct and justify the constructions of a) a line segment congruent to a given line segment; b) the perpendicular bisector of a line segment; c) a perpendicular to a given line from a point not on the line; d) a perpendicular to a given line at a point on the line; e) the bisector of a given angle; f) an angle congruent to a given angle; and g) a line parallel to a given line through a point not on the given line.

HINTS & NOTES

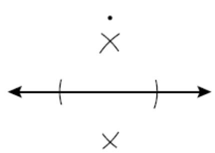
Perpendicular bisector:



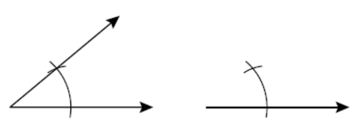
Perpendicular bisector from a point not on the line:



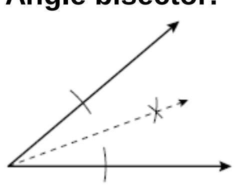
Perpendicular to a line from a point on the line:



Copy an angle:

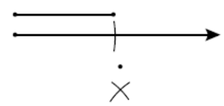


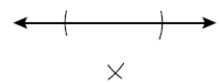
Angle bisector:

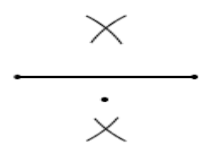


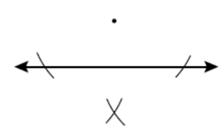
PRACTICE G.4

2. Which drawing shows the arcs for a construction of a perpendicular to a line from a point not on the line?

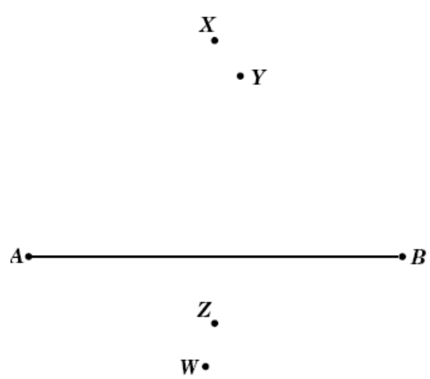
A. 

B. 

C. 

D. 

2.



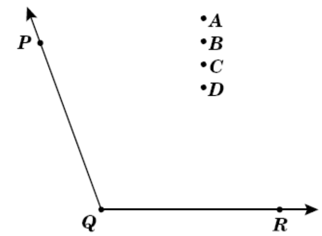
Which pair of points determines the perpendicular bisector of line segment AB?

F. *X*, *W*
G. *X*, *Z*
H. *Y*, *W*
J. *Y*, *Z*

HINTS & NOTES

PRACTICE G.4

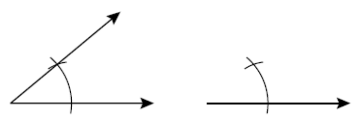
3. Use your compass and straightedge to construct the bisector of this angle.



Which point lies on the bisector?

A. A
B. B
C. C
D. D

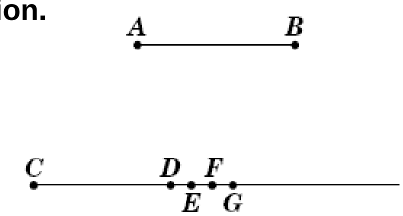
4.



The drawing shows the arcs used to construct -

F. a bisector of a given angle
G. an angle congruent to a given angle
H. a bisector of a given line
J. a perpendicular of a line at a point on the line

5. Use your compass to answer the following question.

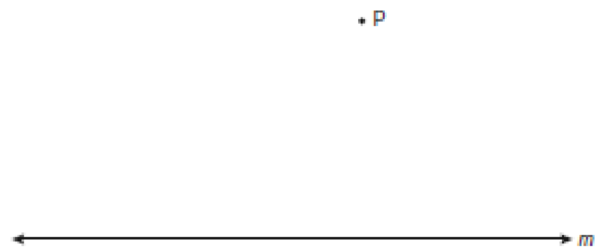


Which line segment is congruent to $\overline{AB}$?

A. $\overline{CD}$
B. $\overline{CE}$
C. $\overline{CF}$
D. $\overline{CG}$

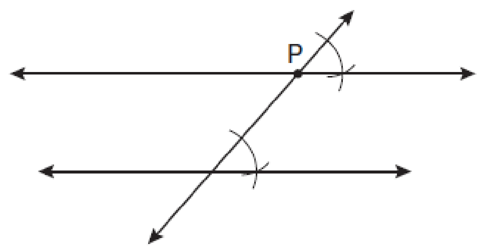
PRACTICE G.4

6. Using a compass and straightedge, construct the perpendicular bisector of Segment AB shown to the right. Show all construction marks.



7. Using a compass and straightedge, construct a line that passes through point P and is perpendicular to line m. [Leave all construction marks.]

8. Which geometric principle is used to justify the construction below?



F. A line perpendicular to one of two parallel lines is perpendicular to the other.
G. Two lines are perpendicular if they intersect to form congruent adjacent angles.
H. When two lines are intersected by a transversal and alternate interior angles are congruent, the lines are parallel.
J. When two lines are intersected by a transversal and the corresponding angles are congruent, the lines are parallel.

SKILLS CHECKLIST: *I can…*

- ☐ Construct and justify the constructions of (a line segment congruent to a given lien segment, the perpendicular bisector of a lien segment, a perpendicular to a give line from a point not on a line, a perpendicular to a given line at a point on the line, the bisector of a given angle, an angle congruent to a given angle and a line parallel to a given line through a point not on the given line.
- ☐ Apply these constructions to possible real world applications to include:
  - Constructing equilateral triangle, square, and regular hexagon inscribed in a circle
  - Constructing inscribed and circumscribed circles of a triangle
  - Constructing a tangent line from a point outside a given circle to the circle

TRIANGLES

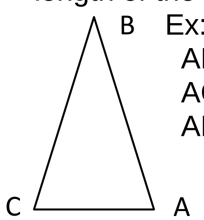
G.5 a-d	1.___2.___3.___4.___5.___6.___ 7.______________________________________ 8.___ 9.___10.___
G.6 & G.7	1.___2.___3.___4.___5.___6.___7.___8.___ 9.___10.___11.___
G.8	1.___2.___3.___4.___5.___6.___7._________ 8.__________9.__________________

SOL G.5

The student, given information concerning the lengths of sides and/or measures of angles in triangles, will a) order the sides by length, given the angle measures; b) order the angles by degree measure, given the side lengths; c) determine whether a triangle exists; and d) determine the range in which the length of the third side must lie. These concepts will be considered in the context of real-world situations.

HINTS & NOTES

Triangle Inequality Theorem:
The sums of the lengths of any two sides of a triangle is greater than the length of the third side.



Ex:
AB + BC > AC
AC + BC > AB
AB + AC > BC

To determine whether a triangle can exist:
Add the 2 smallest sides and that must be greater than the third side

To determine the range of a third side:
The 3rd side must be greater than the

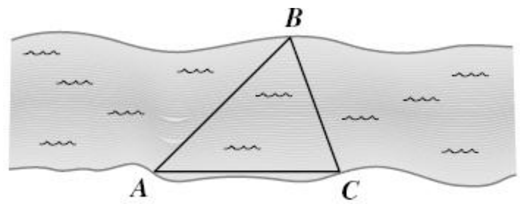
2nd side – 1st side

OR
less than the

1st side + 2nd side

PRACTICE G.5

1. **On the shores of a river, surveyors marked locations, A, B, and C. The measure of $\angle ACB$ = 70°, and the measure of $\angle ABC$ = 65°?**



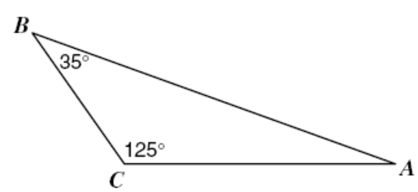
Which lists the distances between these locations in order, greatest to least?

A. A to B, B to C, A to C
B. B to C, A to B, A to C
C. A to B, A to C, B to C
D. A to C, A to B, B to C

2. **Which of the following could be the lengths of the sides of Δ*ABC*?**

F. *AB* = 12, *BC* = 15, *AC* = 2
G. *AB* = 9, *BC* = 15, *CA* = 4
H. *AB* = 150, *BC* = 100, *CA* = 50
J. *AB* = 10, *BC* = 8, *AC* = 12

3. **In the drawing, the measure of ∠ C = 125° and the measure of ∠ B = 35°.**



Which is the shortest side of the triangle?

A. $\overline{AC}$
B. $\overline{AB}$
C. $\overline{EB}$
D. $\overline{BC}$

HINTS & NOTES

Comparing sides and angles of a triangle:
If one side of a triangle is longer than another side, then the angle opposite the longer side is larger than the angle opposite the shorter side and vice-versa.

****Remember****
Biggest angle is opposite the longest side.
Smallest angle is opposite the shortest side.

Make sure that you read what the question is asking for!
It might say to list in order from greatest to least or least to greatest!

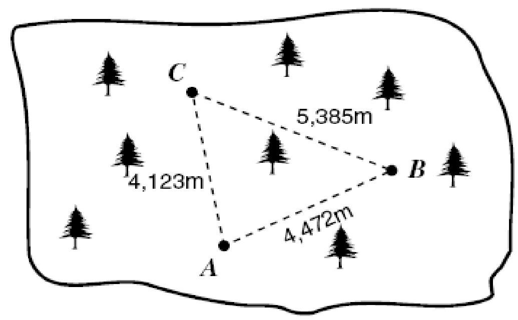
Hinge Theorem - if two sides of one triangle are congruent to two sides of another triangle, and the included angle of the first is larger than the included angle of the second, then the third side of the first triangle is longer than the third side of the second triangle (*Practice on next page.)*

PRACTICE G.5

4. **Sara is building a triangular pen for her pet rabbit. If two of the sides measure 8 feet and 15 feet, the length of the third side could be**

   F. 13 ft
   G. 23 ft
   H. 7 ft
   J. 3 ft

5. **Three lookout towers are located at points A, B, and C on the section of the park in the drawing.**



Which of the following statements is true concerning ΔABC formed by the towers?

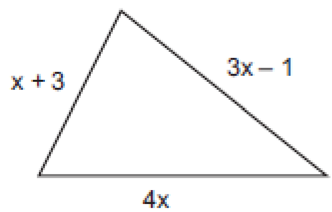
   A. m$\angle A$ is least.
   B. m$\angle A$ is greatest.
   C. m$\angle C$ is least.
   D. m$\angle C$ is greatest.

6. In any ΔXYZ, which statement is always true?

   F. XY + YZ < XZ
   G. XY + YZ > XZ
   H. m$\angle X$ + m$\angle Y$ < 90°
   J. m$\angle X$ + m$\angle Y$ = 90°

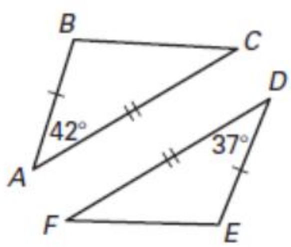
PRACTICE G.5

7. The plot of land illustrated in the accompanying diagram has a perimeter of 34 yards. Find the length, in yards, of *each* side of the figure. Could these measures actually represent the measures of the sides of a triangle? Explain your answer.

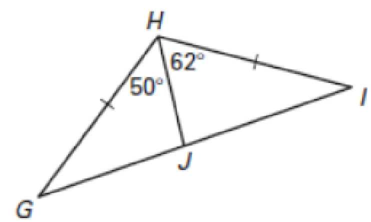


Complete with <, > or =.

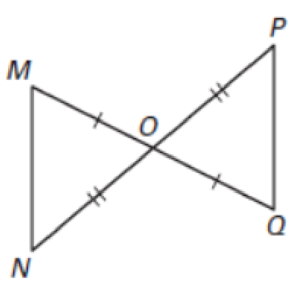
8. BC ____ EF



9. JG ____ JI



10. MN ____ QP



SKILLS CHECKLIST: *I can …*

- ☐ Order sides of a triangle by their lengths when given the measures of the angles.
- ☐ Order the angles of a triangle by their measures when given the lengths of sides.
- ☐ Given the lengths of three segments, determine whether a triangle could be formed.
- ☐ Given the lengths of two sides of a triangle, determine the range in which the length of the third side must lie.
- ☐ Solve real-world problems given information about the lengths of sides and/or measures of angles in triangles.

SOL G.6
The student, given information in the form of a figure or statement, will prove two triangles are congruent, using algebraic and coordinate methods as well as deductive proofs.

SOL G.7
The student, given information in the form of a figure or statement, will prove two triangles are similar, using algebraic and coordinate methods as well as deductive proof.

HINTS & NOTES

To prove triangles congruent:
SSS (Side-Side-Side)
SAS (Side-Angle-Side)
ASA (Angle-Side-Angle)
AAS (Angle-Angle-Side)
HL (Hypotenuse-Leg)

If two triangles share a side **– Reflexive Property.**

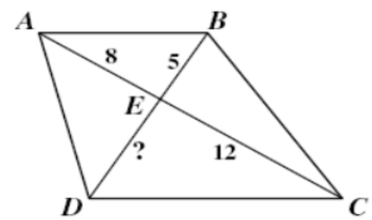
Vertical angles are always congruent.
(Look for an x)

CPCTC – Corresponding Parts of Congruent Triangles are Congruent

To prove triangles are similar:
SAS Similarity
SSS Similarity
AA Similarity

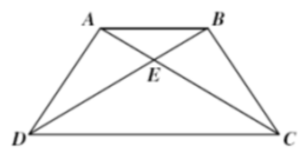
PRACTICE G.6 & G.7

1. **In the figure, AE = 8, CE = 12, and BE = 5. What value for the measure of DE would make ΔABE similar to ΔCDE?**



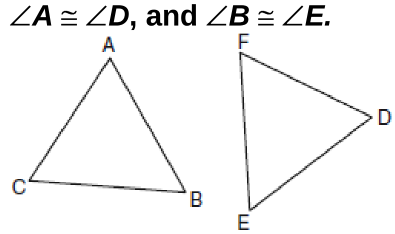
   A. 15
   B. 8
   C. 7.5
   D. 3.3

2. **Given:** $\overline{AC} \cong \overline{BD}$ and $\overline{AD} \cong \overline{BC}$**. Which could be used to prove ΔDCA ≅ ΔCDB?**



   F. (SSS)
   G. (SAS)
   H. (ASA)
   J. (AAS)

3. In the diagram of $\triangle ABC$ **and** $\triangle DEF$ **below,** $\overline{AB} \cong \overline{DE}$ **,** $\angle \boldsymbol{A} \cong \angle \boldsymbol{D}$**, and** $\angle \boldsymbol{B} \cong \angle \boldsymbol{E}$**.**

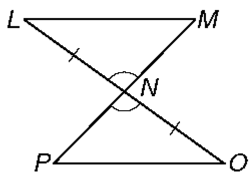


Which method can be used to prove $\triangle ABC \cong \triangle DEF$ **?**

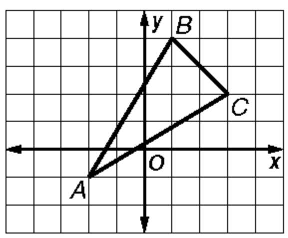
A. SSS
B. SAS
C. ASA
D. HL

PRACTICE G.6 & G.7

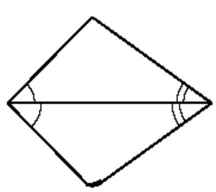
For questions 4 and 5, use this figure.



4. **What additional information is needed to prove that $\triangle MNL$ is congruent to $\triangle PNO$ by ASA?**
   F $\overline{MN} \cong \overline{PN}$
   G $\overline{LM} \cong \overline{OP}$
   H $\angle M \cong \angle P$
   J $\angle L \cong \angle O$
5. **What additional information is needed to prove $\triangle MNL$ congruent to $\triangle PNO$ by SAS?**
   A $\overline{MN} \cong \overline{PN}$
   B $\overline{LM} \cong \overline{OP}$
   C $\angle M \cong \angle P$
   D $\angle L \cong \angle O$
6. **Which are the vertices of a triangle congruent to the triangle in the figure?**



   F $X(4, ^{-}3)$, $Y(^{-}1, 0)$, $Z(1, 2)$
   G $D(5, 0)$, $E(1, 3)$, $F(^{-}1, ^{-}2)$
   H $G(^{-}4, 3)$, $H(^{-}1, 2)$, $I(1, 0)$
   J $P(3, 8)$, $Q(8, 5)$, $R(6, 9)$
7. **Which postulate or theorem shows that the triangles shown below are congruent?**

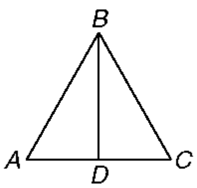


   A SSS
   B SAS
   C ASA
   D HL

PRACTICE G.6 & G.7

For questions 8-11, use this figure and proof.

Given: $\overline{BD}$ bisects $\angle ABC$.
$\triangle ABC$ is isosceles with base $\overline{AC}$.
Prove: $\triangle ABD \cong \triangle CBD$



Statements	**Reasons**
1. $\overline{BD}$ bisects $\angle ABC$.	1. Given
2. $\overline{BD} \cong \overline{BD}$	2. ?
3. $\angle ABD \cong \angle CBD$	3. ?
4. $\overline{AB} \cong \overline{CB}$	4. ?
5. $\triangle ABD \cong \triangle CBD$	5. ?

8. **What is the reason for Statement 2?**
   F Transitive Property of Congruence
   G Reflexive Property of Congruence
   H Symmetric Property of Congruence
   J Given
9. **What is the reason for Statement 3?**
   A If two sides of a triangle are congruent then the angles opposite them are congruent.
   B Reflexive Property of Congruence
   C An angle bisector divides an angle into two congruent angles.
   D An isosceles triangle has two congruent sides.
10. **What is the reason for Statement 4?**
    F If two sides of a triangle are congruent, then the angles opposite them are congruent.
    G An isosceles triangle has two congruent sides.
    H A kite has two congruent sides.
    J Reflexive Property of Congruence
11. **What is the reason for Statement 5?**
    A AAS
    B SAS
    C ASA
    D SSA

SKILLS CHECKLIST: *I can…*

- ☐ Use definitions, postulates, and theorems to prove triangles congruent.
- ☐ Use coordinate methods, such as the distance formula and the slope formula, to prove two triangles are congruent.
- ☐ Use algebraic methods to prove two triangles are congruent.
- ☐ Use definitions, postulates, and theorems to prove triangles similar.
- ☐ Use algebraic methods to prove that triangles are similar.
- ☐ Use coordinate methods, such as the distance formula and the slope formula, to prove two triangles are similar.

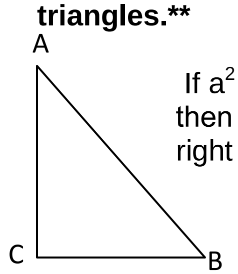
SOL G.8

The student will solve real-world problems involving right triangles by using the Pythagorean Theorem and its converse, properties of special right triangles, and right triangle trigonometry.

HINTS & NOTES

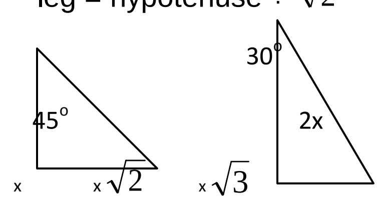
Pythagorean Theorem:
$a^2 + b^2 = c^2$, where c is the hypotenuse

****You can only use the Pythagorean theorem on RIGHT triangles.****



If $a^2 + b^2 = c^2$, then ΔABC is a right triangle.

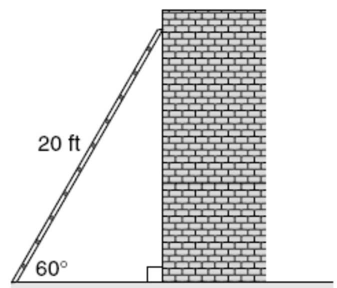
Special Right Triangles:
45^o-45^o-90^o
2 legs are congruent
2 angles opposite the legs are congruent and = 45^o
Formulas:
leg = leg
hypotenuse = leg x $\sqrt{2}$
leg = hypotenuse ÷ $\sqrt{2}$



30^o-60^o-90^o
Formulas:
long leg = short leg x $\sqrt{3}$
hypotenuse = short leg x 2
short leg = long leg ÷ $\sqrt{3}$
short leg = hypotenuse ÷ 2

PRACTICE G.8

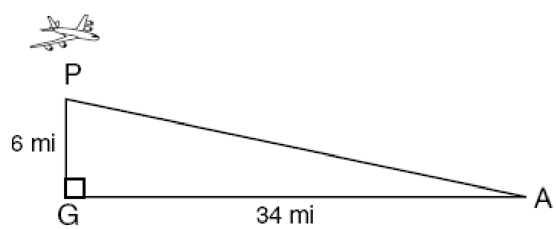
1.



A 20-foot ladder leaning against a building makes an angle of 60^o with the ground. How far from the base of the building is the foot of the ladder?

A. 17.3 ft
B. 10 ft
C. 8.2 ft
D. 5 ft

2.



An airplane is 34 ground miles from the end of a runway (GA) and 6 miles high (PG) when it begins its approach to the airport. To the nearest mile, what is the distance (PA) from the airplane to the end of the runway?

F. 35 mi
G. 37 mi
H. 39 mi
J. 41 mi

HINTS & NOTES

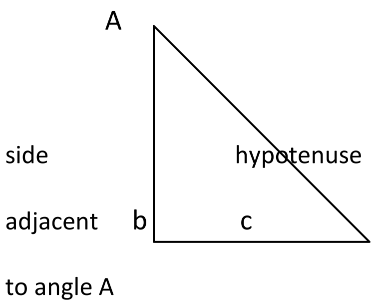
Trigonometric Ratios:

SOH-CAH-TOA

sin x = $\frac{opposite}{hypotenuse}$

cos x = $\frac{adjacent}{hypotenuse}$

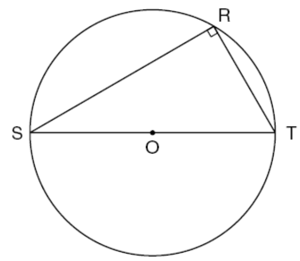
tan x = $\frac{opposite}{adjacent}$



*****Use $\sin^{-1}$, $\cos^{-1}$, $\tan^{-1}$ to find angles measures!**

PRACTICE

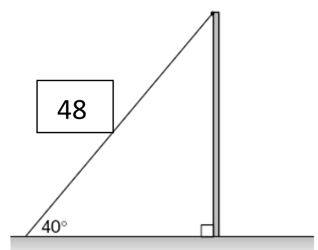
3.



In circle O, $\angle$ RST formed by chord RS and diameter ST has a measure of 30°. If the diameter is 18 centimeters, what is the length of chord SR?

A. $18\sqrt{3}$
B. $18\sqrt{2}$
C. $9\sqrt{3}$
D. $9\sqrt{2}$

4.



A cable 48 feet long stretches from the top of a pole to the ground. If the cable forms a 40° angle with the ground, which is closest to the height of the pole?

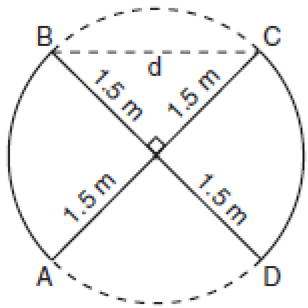
F. 40.3 ft
G. 36.8 ft
H. 30.9 ft
J. 26.4 ft

5. Which set of numbers does *not* represent the sides of a right triangle?

A. 6, 8, 10
B. 8, 15, 17
C. 8, 24, 25
D. 15, 36, 39

PRACTICE G.8 CONTINUED

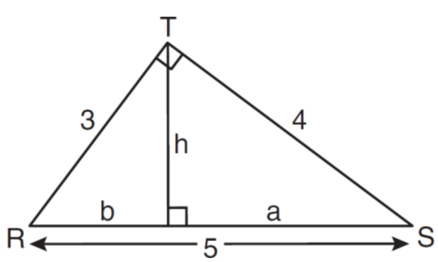
6. An overhead view of a revolving door is shown in the accompanying diagram. Each panel is 1.5 meters wide.



What is the approximate width of d, the opening from B to C?

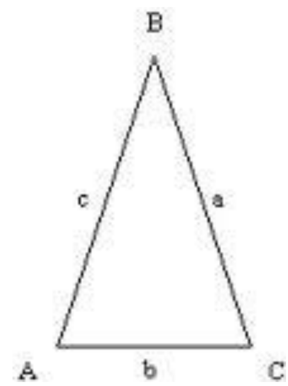
F. 1.50 m
G. 1.73 m
H. 3.00 m
J. 2.12 m

7. In the diagram below, !*RTS* is a right triangle. The altitude, h, to the hypotenuse has been drawn. Determine the length of h.



with sides lengths 7, 24, 29 is right, acute, or obtuse.

9.



In the isosceles triangle above a = c = 10, and b = 6. Find the measures of angles *A, B* and *C.*

SKILLS CHECKLIST: *I can…*

- ☐ Determine whether a triangle formed with three given lengths is a right triangle.
- ☐ Solve for missing lengths in geometric figures, using properties of $45^\circ - 45^\circ - 90^\circ$ triangles.
- ☐ Solve for missing lengths in geometric figures, using properties of $30^\circ - 60^\circ - 90^\circ$ triangles.
- ☐ Solve problems involving right triangles, using sine, cosine, and tangent ratios.
- ☐ Solve real-world problems, using right triangle trigonometry and properties of right triangles.
- ☐ Explain and use the relationship between the sine and cosine of complementary angles.

Polygons, Circles and Three-Dimensional Figures

G.9	1.___ 2.___3.____4.____5.____ 6.______________________________ 7.______________________________ 8.______________________________
G.10	1.___2.___3.___4.___5.___ 6.______________________________ 7.______________________________ 8.______________________________
G.11a-c	1.___2.___3.___4.___5.___6.___7.___ 8.______________________________ 9.______________________________
G.12	1.___2.___3.___4.______________5.________________ _ 6._________________________
G.13	1.___2.___3.___4.___5.___6.___
G.14a-d	

SOL G.9

The student will verify characteristics of quadrilaterals and use properties of quadrilaterals to solve real-world problems.

HINTS & NOTES

Quadrilateral – polygon with four sides

Properties:

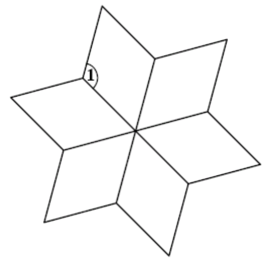
Parallelogram:
- Opposite sides $\cong$
- Opposite sides parallel
- Opposite angles $\cong$
- Consecutive angles are supplementary
- Diagonals bisect each other

Rectangle:
- Opposite sides $\cong$
- Opposite sides parallel
- Opposite angles $\cong$
- Consecutive angles are supplementary
- Diagonals bisect each other
- Four right angles (90^o)
- Diagonals are $\cong$

Rhombus:
- Opposite sides $\cong$
- Opposite sides parallel
- Opposite angles $\cong$
- Consecutive angles are supplementary
- Diagonals bisect each other
- Four $\cong$ sides
- Diagonals are perpendicular

PRACTICE G.9

1.



The design for a quilt piece is made up of 6 congruent parallelograms. What is the measure of $\angle$ 1?

A. 150^o
B. 120^o
C. 60^o
D. 30^o

2.

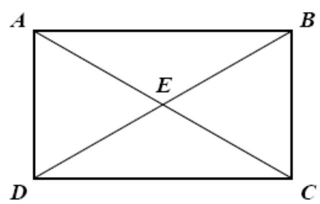


Figure *ABCD* is a rectangle. $\overline{AC}$ and $\overline{BD}$ are diagonals. AC = 30 meters and BC = 18 meters. What is the length of $\overline{DE}$?

F. 8 m
G. 10 m
H. 15 m
J. 24 m

3. Which of the following quadrilaterals could have diagonals that are congruent but do *not* bisect each other?

A. A rectangle **B.** A rhombus
C. A trapezoid **D.** A parallelogram

HINTS & NOTES

Square:
- Opposite sides ≅
- Opposite sides parallel
- Opposite angles ≅
- Consecutive angles are supplementary
- Diagonals bisect each other
- Four right angles
- Four ≅ sides
- Diagonals are ≅
- Diagonals are perpendicular

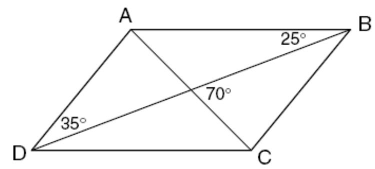
Kite:
- Diagonals are perpendicular
- One pair of opposite angles are ≅

Trapezoid:
- One pair of opposite sides are parallel
- Two pairs of consecutive interior angles are supplementary

Isosceles trapezoid:
- One pair of opposite sides are parallel
- Two pairs of consecutive interior angles are supplementary
- One pair of opposite sides parallel
- Two pairs of consecutive angles are congruent
- Diagonals bisect each other

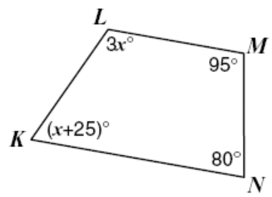
PRACTICE G.9

4. In parallelogram *ABCD*, what is m∠ *DBC*?



A. 25^o
B. 35^o
C. 45^o
D. 70^o

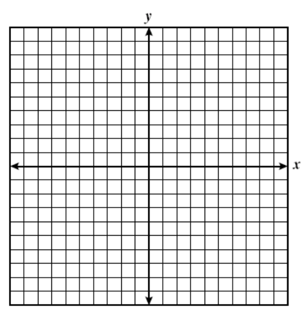
5.



Given quadrilateral *KLMN*, what is the value of *x*?

F. 35
G. 40
H. 45
J. 50

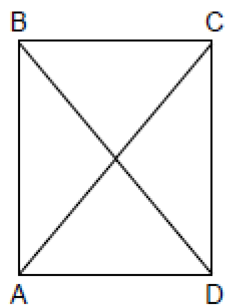
6. Three vertices of a parallelogram have coordinates (0,1), (3,7), and (4,4).



Place a point on the graph that could represent the fourth vertex of the parallelogram.

PRACTICE G. 9

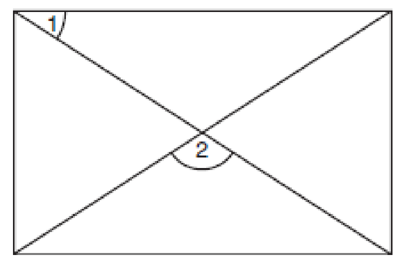
7. In the accompanying diagram of rectangle ABCD, $m\angle BAC = 3x + 4$ and $m\angle ACD = x + 28$.



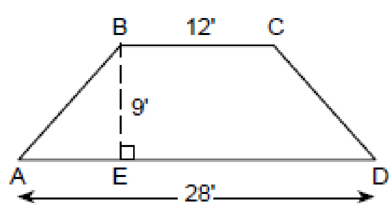
What is $m\angle CAD$?

8. As shown in the accompanying diagram, a rectangular gate has two diagonal supports.

If $m\angle 1 = 42$, what is $m\angle 2$?



1. The cross section of an attic is in the shape of an isosceles trapezoid, as shown in the accompanying figure. If the height of the attic is 9 feet, *BC* = 12 feet, and *AD* = 28 feet, find the length of *AB* to the nearest foot.



SKILLS CHECKLIST: *I can…*

- ☐ Solve problems, including real-world problems, using the properties specific to parallelograms, rectangles, rhombi, squares, isosceles trapezoids, and trapezoids.
- ☐ Prove that quadrilaterals have specific properties, using coordinate and algebraic methods, such as the distance formula, slope, and midpoint formula.
- ☐ Prove the characteristics of quadrilaterals, using deductive reasoning, algebraic, and coordinate methods.
- ☐ Prove properties of angles for a quadrilateral inscribed in a circle.

SOL G.10

The student will solve real-world problems involving angles of polygons.

HINTS & NOTES

Polygon	**# of sides**	**Sum of the interior angles**
Triangle	3	180^o
Quadrilateral	4	360^o
Pentagon	5	540^o
Hexagon	6	720^o
Heptagon	7	900^o
Octagon	8	1080^o
Nonagon	9	1260^o
Decagon	10	1440^o

Sum of the exterior angles of any polygon is always = 360^o.

Remember: Number of sides = number of interior angles

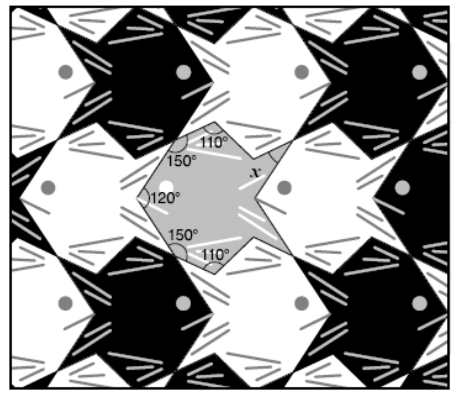
Equilateral – means all sides are equal

Equiangular - means all angles are congruent.

Regular – means all sides are equal in length and all angles are equal in measure

PRACTICE G.10

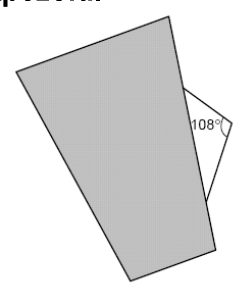
1. **Some of the angle measures are given for one of the fish-shaped polygons in this tessellation.?**



What is the value of x?

A. 30^o
B. 40^o
C. 45^o
D. 60^o

2. **In the drawing a regular polygon is partially covered by the trapezoid.**



How many sides does the covered polygon have?

F. 8
G. 6
H. 5
J. 4

HINTS & NOTES

Formulas:
(n = number of sides)

Sum of the interior angles:
$(n-2)180^o$

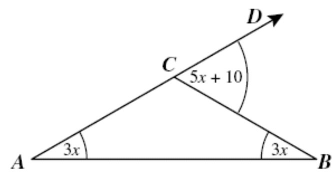
Each interior angle:
$\frac{(n-2)180}{n}$

Sum of the exterior angles:
360^o

Each exterior angle:
$\frac{360}{n}$

PRACTICE G.10

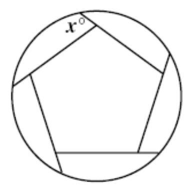
3. The figure has angle measures as shown.



What is the measure of $\angle$ BCD?

A. 30^o
B. 60^o
C. 80^o
D. 120^o

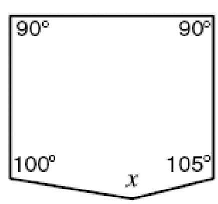
4.



A floor tile is designed with a regular pentagon in the center of the tile with its sides extended. What is the value of x?

F. 72^o
G. 90^o
H. 110^o
J. 120^o

5.

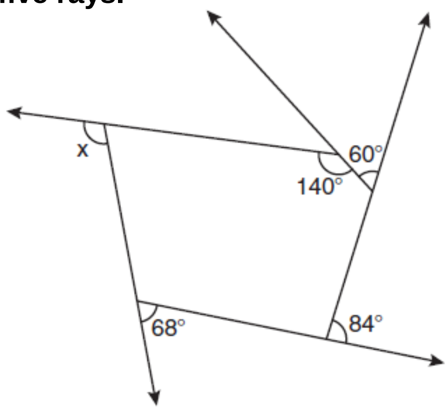


What is the value of x in the pentagon above?

A. 90^o
B. 155^o
C. 245^o
D. 335^o

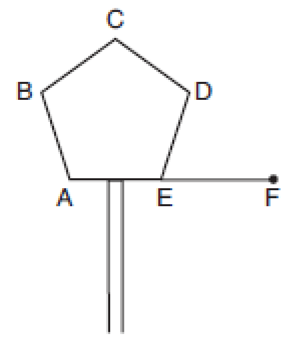
PRACTICE G.10

6. The pentagon in the diagram below is formed by five rays.



What is the degree measure of $\angle x$?

7. One piece of the birdhouse that Natalie is building is shaped like a regular pentagon, as shown in the accompanying diagram.



If side AE is extended to point F, what is the measure of exterior angle DEF?

8. What is the measure of an interior <u>and</u> exterior angle of a regular octagon?

SKILLS CHECKLIST: *I can…*

- ☐ Solve real-world problems involving the measures of interior and exterior angles of polygons.
- ☐ Identify tessellations in art, construction, and nature.
- ☐ Find the sum of the measures of the interior and exterior angles of a convex polygon.
- ☐ Find the measure of each interior and exterior angle of a regular polygon.
- ☐ Find the number of sides of a regular polygon, given the measures of interior or exterior angles of the polygon.

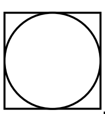
SOL G.11

The student will use angles, arcs, chords, tangents, and secants to a) investigate, verify, and apply properties of circles; and c) find arc lengths and areas of sectors in circles.

HINTS & NOTES

A **circle** always measures 360°.
A **semi-circle** always measures 180°.
Radius – a connects the center of a circle to any point on the circle
Radius = ½ diameter
Diameter – a segment that connects any two points on a circle and passes through the center
Diameter = twice the radius
A **diameter** cuts a circle in half (makes two semi-circles)
Chord – a segment that connects two points on a circle
Tangent – a line, segment, or ray that intersects (touches) a circle in one place

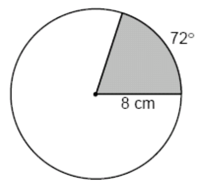
Inscribed – means inside
Ex: a circle inscribed in a square



Circumscribed – means outside
Ex: the square is circumscribed about the circle

PRACTICE G.11

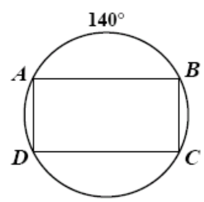
1.



A circle has a radius of 8 centimeters. The measure of the arc of the shaded section is 72°. Which is closest to the area of the shaded section of the circle?

- **A.** 160.8 cm^2
- **B.** 50.3 cm^2
- **C.** 40.2 cm^2
- **D.** 10.1 cm^2

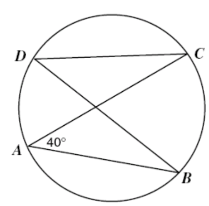
2.



Rectangle *ABCD* is inscribed in a circle. If the measure of the arc *AB* is 140°, what is the measure of arc *BC*?

- **F.** 30°
- **G.** 40°
- **H.** 60°
- **J.** 80°

3.



If m∠ *CAB* = 40°, what is m∠ *CDB*?

- **A.** 20°
- **B.** 40°
- **C.** 60°
- **D.** 80°

HINTS and NOTES

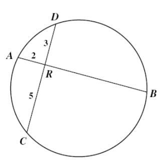
Arc – a piece of a circle

Angle Formulas:

- Tangent $\perp$ radius
- Central angle = intercepted arc
- Inscribed angle = ½ intercepted arc
- Outside angle = ½ (big arc – little arc)
- Inside angle (makes an x) = ½ (intercepted arc + intercepted arc)
- Magic hat (formed by two tangents to a circle) : top of the head + tip of the hat = 180^o and sides are congruent

When two chords intersect in a circle (make an x) : then part x part = part x part

Ex:

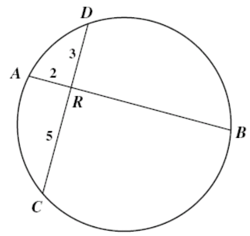


AR x RB = CR x RD

When a secant intersects another secant outside of the circle:
WO = WO
Whole x outside part = whole x outside part

PRACTICE G.11

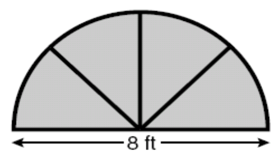
4.



Chords $\overline{AB}$ and $\overline{CD}$ intersect at *R*. Using the values shown in the diagram, what is the measure of *RB*?

F. 6
G. 7.5
H. 8
J. 9.5

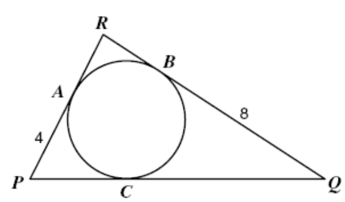
5. This is a sketch of a stained-glass window in the shape of a semicircle.



Ignoring the seams, how much glass is needed for the window?

A. 4π sq ft
B. 8π sq ft
C. 12π sq ft
D. 16π sq ft

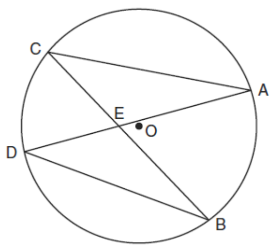
6.



A, B, and C are points of tangency. AP = 4 and BQ = 8. What is the measure of $\overline{PQ}$?

F. 4
G. 8
H. 12
J. $\sqrt{32}$

7. In the diagram below of circle O, chords $\overline{AD}$ and $\overline{BC}$ intersect at E.

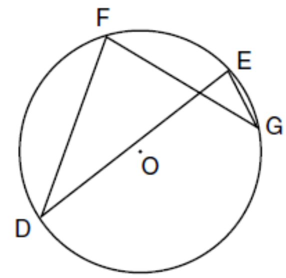


Which relationship must be true?

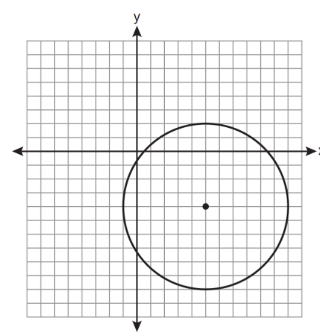
A. !*CAE* ≅ !*DBE*
B. !*AEC* ≅ !*BED*
C. $\angle$*ACB* ≅ $\angle$*CBD*
D. $CA \cong DB$

8. In the diagram below of circle O, chords $\overline{DF}$, $\overline{DE}$, $\overline{FG}$, and $\overline{EG}$ are drawn such that $mDF : mFE : mEG : mGD$ = 5: 2: 1: 7.

Identify one pair of inscribed angles that are congruent to each other and give their measure.



9. Write an equation of the circle graphed in the diagram below.



SKILLS CHECKLIST: *I can…*

- ☐ Find lengths, angle measures, and arc measures associated with two intersecting chords, two intersecting secants, an intersecting secant and tangent, two intersecting tangents and central and inscribed angles.
- ☐ Calculate the area of a sector and the length of an arc of a circle, using proportions.
- ☐ Solve real-world problems associated with circles, using properties of angles, lines, and arcs.
- ☐ Verify properties of circles, using deductive reasoning, algebraic, and coordinate methods.

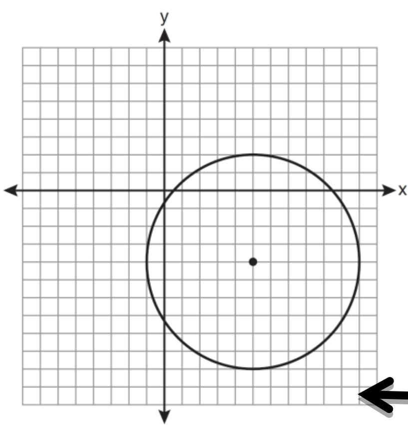
SOL G.12

The student, given the coordinates of the center of a circle and a point on the circle, will write the equation of the circle.

HINTS and NOTES

Circle- locus of points equidistant from a given point (center)

Standard Form Equation of Circle-
$(x-h)^2+(y-k)^2=r^2$
Coordinates of the center of the circle are (h, k) and r is the length of the radius.



PRACTICE G.12

1. **Write the standard equation of a circle with its center at the origin and radius 7.**

   A. $x^2+y^2=49$
   B. $x^2+y^2=14$
   C. $\frac{x^2}{14}+\frac{y^2}{14}=1$
   D. $x^2+y^2=7$

2. **Write the standard equation of a circle with center (4, –4) and radius 4.**

   F. $(x-4)^2+(y+4)^2=16$
   G. $(x+4)^2+(y-4)^2=4$
   H. $(x+4)^2-(y-4)^2=4$
   J. $(x-4)^2+(y-4)^2=16$

3. **The standard equation of a circle with center (–4, 3) and radius 7 is _____.**

   A. $(x-4)^2+(y+3)^2=7$
   B. $(x-4)^2+(y+3)^2=49$
   C. $(x+4)^2+(y-3)^2=49$
   D. $(x+4)+(y-3)=7$

4. **Identify the center, radius and diameter of a circle with the following standard equation.**

$$(x-4)^2+(y+10)^2=100$$

5. **Write the standard equation of a circle with center (-2, 5) and the point (4, 13) on the circle.**

6. **Write an equation of the circle graphed in the diagram graphed to the left.**

SKILLS CHECKLIST: *I can…*

- ☐ Identify the center, radius, and diameter of a circle from a given standard equation.
- ☐ Use the distance formula to find the radius of a circle.
- ☐ Given the coordinates of the center and radius of the circle, identify a point on the circle.
- ☐ Given the equation of a circle in standard form, identify the coordinates of the center and find the radius of the circle.
- ☐ Given the coordinates of the endpoints of a diameter, find the equation of the circle.
- ☐ Given the coordinates of the center and a point on the circle, find the equation of the circle.
- ☐ Recognize that the equation of a circle of a given center and radius is derived using the Pythagorean Theorem.

SOL G.13

The student will use formulas for surface area and volume of three-dimensional objects to solve real-world problems.

HINTS & NOTES

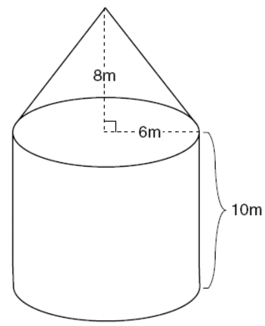
*****Use the formula sheet provided for these problems*****

Surface area (can also be called total area)– the amount of something that it takes to cover a figure completely
-measured in square units

Volume (can also be called capacity) – the amount of space you have to fill something
-measured in cubic units

PRACTICE G.13

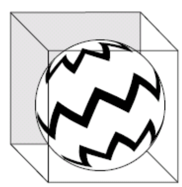
1.



Rounded to the nearest hundred cubic meters, what is the total capacity (cone and cylinder) of the storage container?

- **A.** 1,400
- **B.** 2,000
- **C.** 5,700
- **D.** 8,100

2.

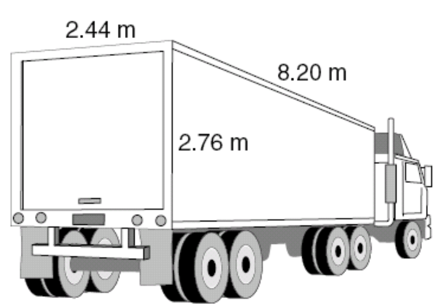


A sphere with a 2-inch radius is packed in a cube so that all sides touch. How much empty space is left in the cube?

- **F.** 17.8 cu in.
- **G.** 30.5 cu in.
- **H.** 33.5 cu in.
- **J.** 47.25 cu in.

3. The cargo space of the truck is 2.44 meters wide, 2.76 meters high, and 8.20 meters long. How many cubic meters of cargo space does the truck have?

- **A.** 26.80
- **B.** 55.22
- **C.** 98.75
- **D.** 110.44



PRACTICE G.13

5. A spherical paintball measures 1.5 centimeters in diameter. Approximately how much paint is in it?

- **F.** 14.13 cm^3
- **G.** 9.42 cm^3
- **H.** 7.07 cm^3
- **J.** 1.77 cm^3

6. To the nearest gallon, what is the volume of a cylindrical water heater 1.4 feet in diameter and 4 feet tall?

(1 cubic foot = 7.48 gallons)

- **A.** 34 gal
- **B.** 46 gal
- **C.** 59 gal
- **D.** 132 gal

SKILLS CHECKLIST: *I can…*

- ☐ Find the total surface area of cylinders, prisms, pyramids, cones, and spheres, using the appropriate formulas.
- ☐ Calculate the volume of cylinders, prisms, pyramids, cones, and spheres, using the appropriate formulas.
- ☐ Solve problems, including real-world problems, involving total surface area and volume of cylinders, prisms, pyramids, cones, and spheres as well as combinations of three-dimensional figures.

SOL G.14

The student will use similar geometric objects in two-or three dimensions to a) compare ratios between side lengths, perimeters, areas, and volumes; b) determine how changes in one or more dimensions of an object affect area and /or volume of the object; c) determine how changes in area and /or volume of an object affect one or more dimensions of the object; and d) solve real-world problems about similar geometric objects.

HINTS & NOTES

Dimensional changes:

height length width radius diameter perimeter circum- ference	area	volume
a	a^2	a^3

When finding a missing side of similar figures always set up a proportion keeping corresponding parts in the same order. Cross multiply to solve.

Scale factor:

Ratio of the sides $\rightarrow \frac{a}{b}$

Ratio of the perimeters = scale factor $\rightarrow \frac{a}{b}$

Ratio of the areas =

(scale factor)$^2 \rightarrow \frac{a^2}{b^2}$

Ratio of the volumes =

(scale factor)$^3 \rightarrow \frac{a^3}{b^3}$

PRACTICE G.14

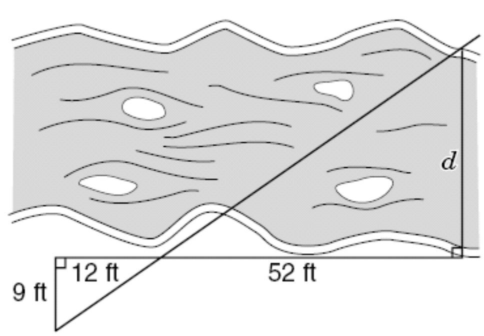
1. **A cylindrical paint can has the capacity of one gallon. For another size can, the height is doubled. What is the capacity of the larger size?**
   A. 2 gal.
   B. 4 gal.
   C. 8 gal.
   D. 16 gal.

2. **If the edge of a cube is 2x, the volume of the cube is:**
   F. $8x^3$
   G. $4x^3$
   H. $2x^3$
   J. $4x^2$

3. **If the radius of a circle is tripled, then the area of the circle is multiplied by:**
   A. 27
   B. 9
   C. 3
   D. 6

4. **Two similar cones have heights 5 and 20. What is the ratio of their volumes?**
   F. 1:16
   G. 1:64
   H. 1:4
   J. 4:16

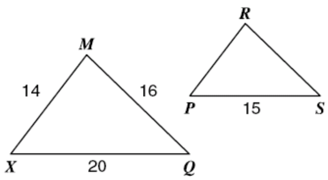
5. **The distance across a river was estimated by making the measurements shown. Which is a good estimate of the distance *d*?**
   A. 9 ft
   B. 36 ft
   C. 39 ft
   D. 69 ft



PRACTICE G.14

6. **The ratio of the circumference of two circles is $\frac{3}{2}$. The radius of the smaller circle is 8 inches. What is the radius of the larger circle?**
   - **F.** $5\frac{1}{3}$ inches
   - **G.** 6 inches
   - **H.** 9 inches
   - **J.** 12 inches

7. **Which proportion can be used to find the value of $\overline{PR}$ if ΔXMQ is similar to ΔPRS?**



   - **A.** $\frac{20}{15} = \frac{14}{PR}$
   - **B.** $\frac{10}{5} = \frac{7}{PR}$
   - **C.** $\frac{14}{20} = \frac{15}{PR}$
   - **D.** $\frac{15}{20} = \frac{14}{PR}$

8. **The ratio between the volumes of two spheres is 27 to 8. What is the ratio between their respective radii?**
   - **F.** 81:64
   - **G.** 27:16
   - **H.** 9:8
   - **J.** 3:2

9. **A triangle has sides whose lengths are 5, 12, and 13. A similar triangle could have sides with lengths of**
   - **A.** 6, 8, and 10
   - **B.** 3, 4, and 5
   - **C.** 7, 24, and 25
   - **D.** 10, 24 and 26

10. **Two triangles are similar. The lengths of the sides of the smaller triangle are 3, 5, and 6, and the length of the longest side of the larger triangle is 18. What is the perimeter of the larger triangle?**
    - **F.** 14
    - **G.** 18
    - **H.** 24
    - **J.** 42

11. **The ratio of the corresponding sides of two similar squares is 1 to 3. What is the ratio of the area of the smaller square to the area of the larger square?**
    - **A.** 1: $\sqrt{3}$
    - **B.** 1: 3
    - **C.** 1: 6
    - **D.** 1: 9

12. **Describe what happens to the volume of a cone if its radius is doubled while its height is halved. The volume is _____.**
    - **F.** unchanged
    - **G.** doubled
    - **H.** increased by a factor of $\frac{1}{3}$
    - **J.** not able to be determined

SKILLS CHECKLIST: *I can…*

- ☐ Compare ratios between side lengths, perimeters, areas, and volumes, given two similar figures.
- ☐ Describe how changes in one or more dimensions affect other derived measures (perimeter, area, total surface area, and volume) of an object.
- ☐ Describe how changes in one or more measures (perimeter, area, total surface area, and volume) affect other measures of an object.
- ☐ Solve real world problems involving measured attributes of similar objects